Testing models with models: The case of game theory

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Roadmap

- Models in Traditional Game Theory
  - The theory
  - Objections
- "Foundational programs"
  - Evolutionary game theory
  - Epistemic game theory
- Philosophical Questions

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Prisoner's dilemma

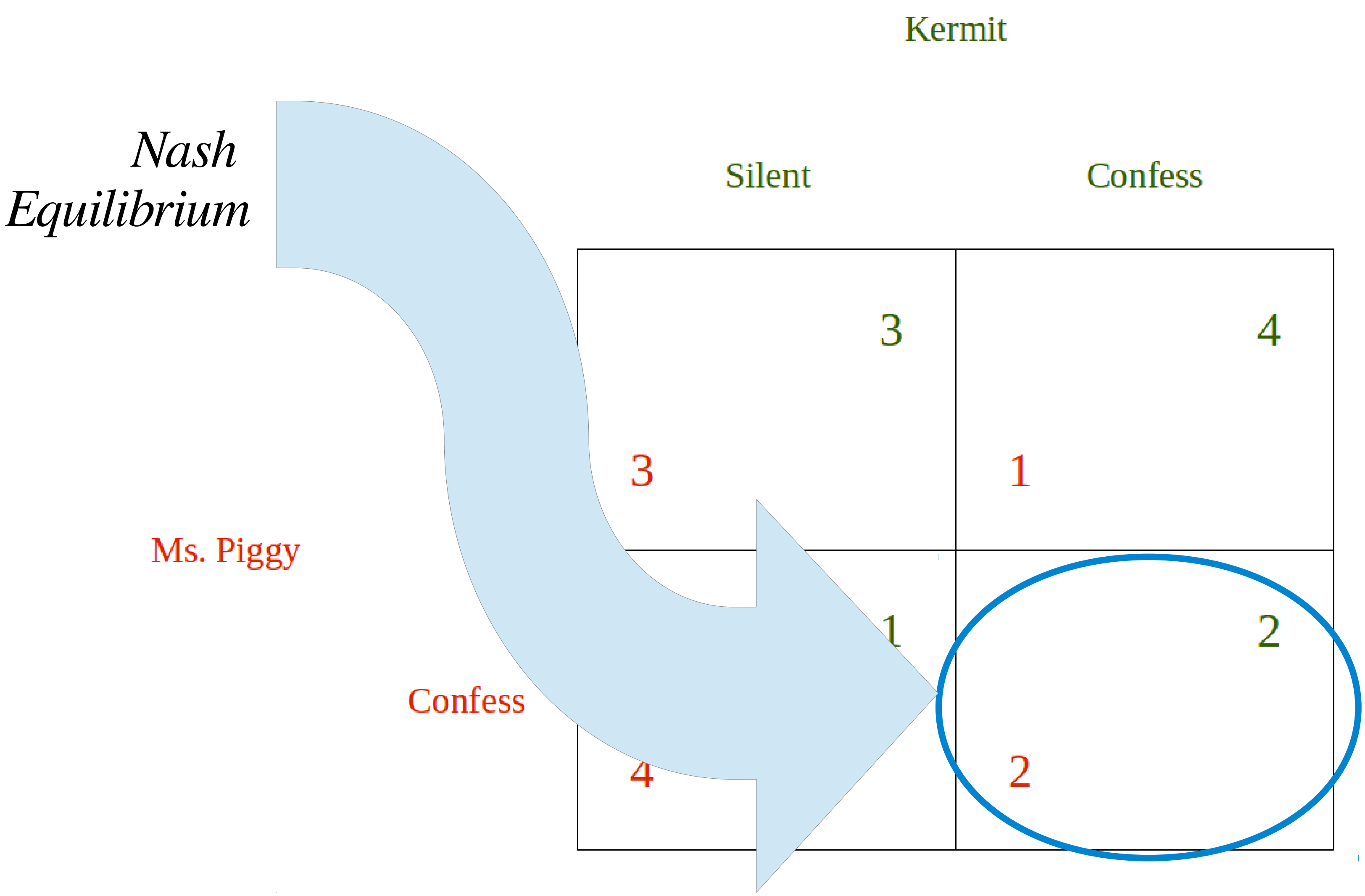
(c) Michelle O'Connell

- Two people arrested
- Each given the chance to confess
- Cannot communicate

Prisoner's dilemma

		Kermit	
		Silent	Confess
Ms. Piggy	Silent	3 3	4 1
	Confess	1 4	2 2

Prisoner's dilemma



Many other games

- Coordination
- Zero sum
- Everything in between

Nash equilibrium

A vector describing a plan of action for each player is a *Nash equilibrium*

if and only if

Each player is doing the best she can given what all the other players are doing

Prediction

- In a game, game theory predicts that everyone will be playing a Nash equilibrium
- Every game has at least one (with randomization)
- Many games have more

Process

- *Step 1*: Find a phenomena of interest
- *Step 2*: Build a game which models that phenomena
- *Step 3*: Find the Nash equilibria of that game
- *Step 4*: Predict that the phenomena will be in the Nash equilibrium

Refinements

- Nash
- Subgame perfection
- Perfect Bayes Nash
- Risk dominance

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Non-concerns

- People care about more than money
- People aren't selfish

Real concerns

- Do people play Nash equilibria?
- Do players always think through the game carefully?
- Do players know that others think through the game carefully?

Illustration

- N-player game
- Everyone guesses a number in [0,100]
- Let m = the average of all the guesses
- The people who guess closest to $(2/3) \times m$ splits a prize
- Everyone else loses

Game theorist reasoning

- Don't guess more than 66.66
  - Suppose everyone knows that
- Don't guess more than 44.44
  - Suppose everyone knows that
- Don't guess more than 29.63
  - Suppose everyone knows that

…

Game theory versus reality

- *Nash equilibrium*: Everyone guess 0
  - Everybody wins.
  - No one can improve.
- No other Nash equilibrium
  - At least one person can improve by changing
- But that's not the outcome

Falsified

- Guess 2/3 of the average isn't the only trouble case
- Some suggest this is the end of the road for game theory

Two observations

- Suppose the game was played repeatedly
  - Guesses would go lower

- Suppose everyone was capable of thinking all the way through and everyone knew it
  - Everyone would guess 0

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Evolutionary game theory

- Suppose a group of people are put together to repeatedly play a game

- After each play they “revise” their strategy to try and improve how they did

“Revision” protocol

- Evolution via natural selection
- Experimentation and adjustment
- Reinforcement of more effective action
- Differential imitation
- “Learning” and myopic best-response

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Replicator dynamics

$$\dot{x}_i = x_i\left(u\left(i,\bar{x}\right) - u\left(\bar{x},\bar{x}\right)\right)$$

- $\dot{x}_i$: Change in proportion playing strategy *i*
- x_i: Proportion playing strategy *i*
- $u(i,\bar{x})$: Payoff of playing *i* against the population (represented by $\bar{x}$)
- $u(\bar{x},\bar{x})$: Average payoff of the population against itself

Replicator dynamics

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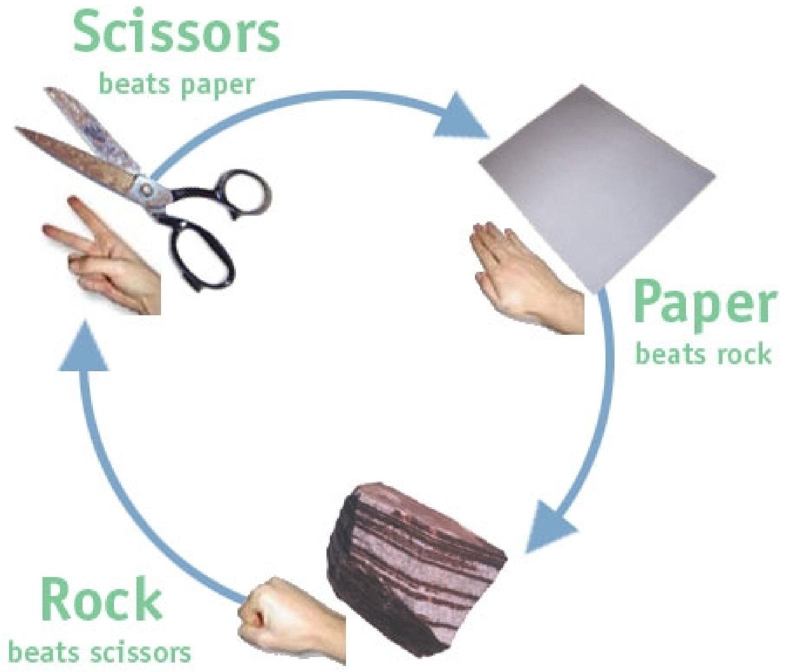
- $\dot{x}_i$: Change in proportion playing strategy *i*
- x_i: Proportion playing strategy *i*
- $\left(u\left(i,\bar{x}\right) - u\left(\bar{x},\bar{x}\right)\right)$: How much better/worse is *i* doing relative to the population

Does this justify Nash?

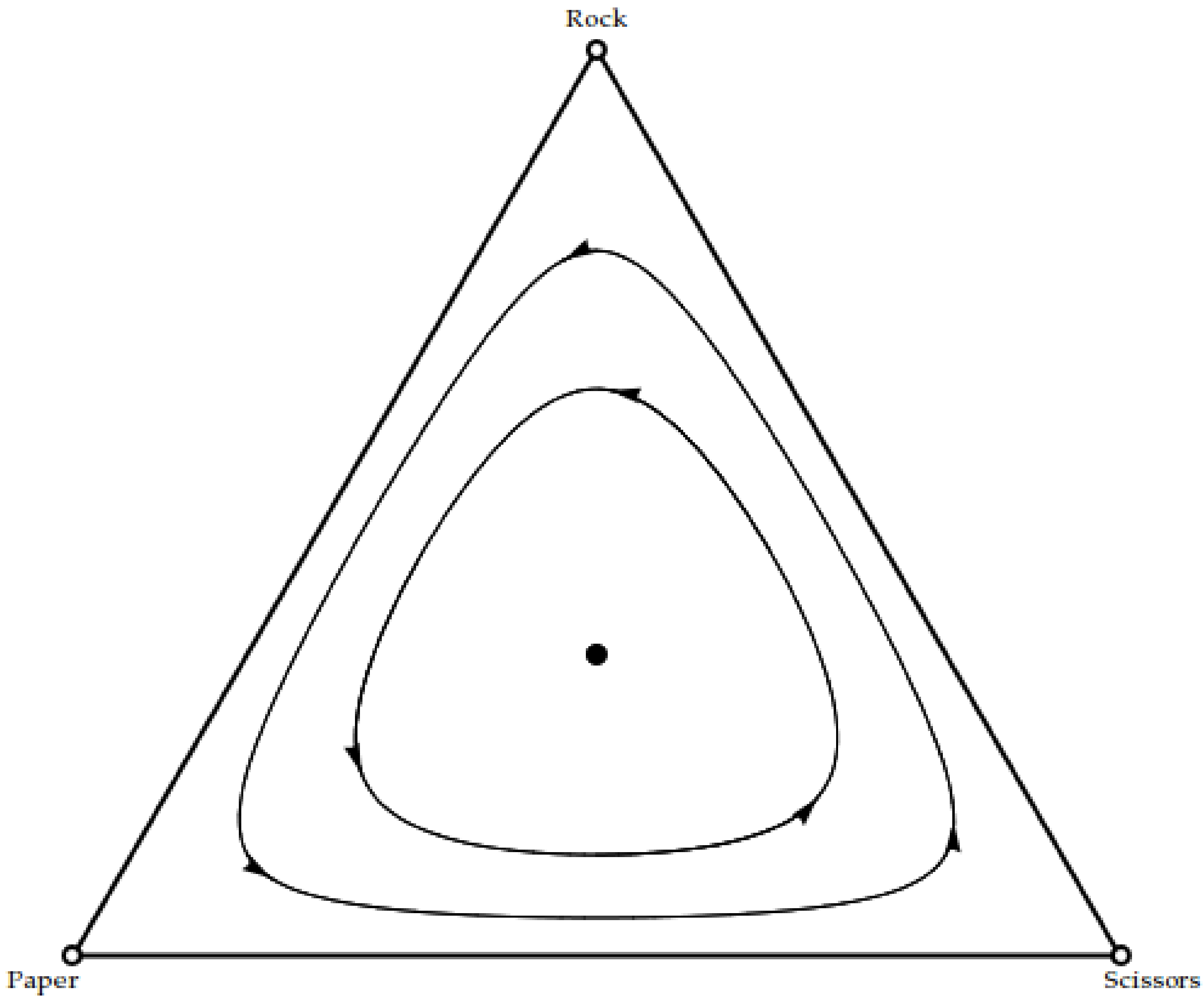
- In two-player two-strategy games: yes
  - Any population which starts with both types ends up in a Nash equilibrium
- In other games: it depends
  - In some games, all populations end in a Nash
  - In some games none do.

Example

	R	P	S
R	0	-1	1
P	1	0	-1
S	-1	1	0

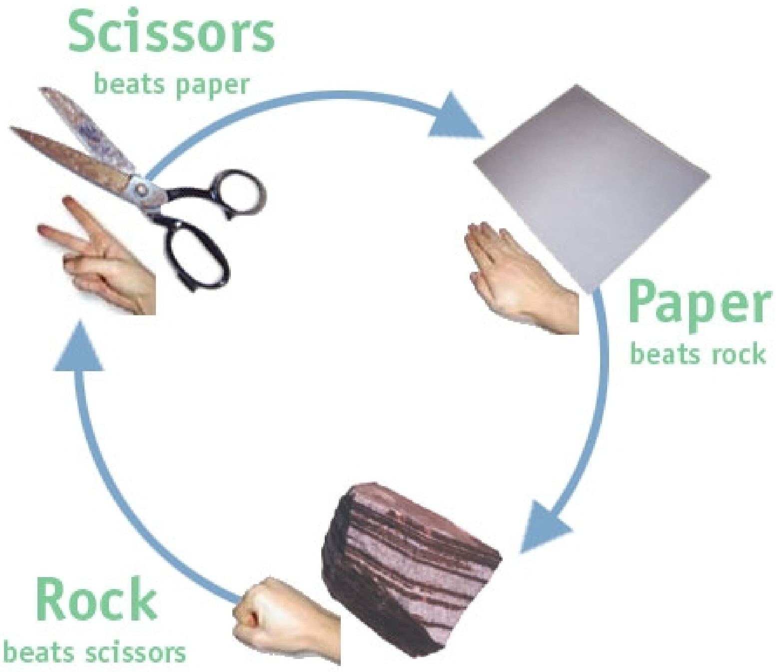


Never Nash

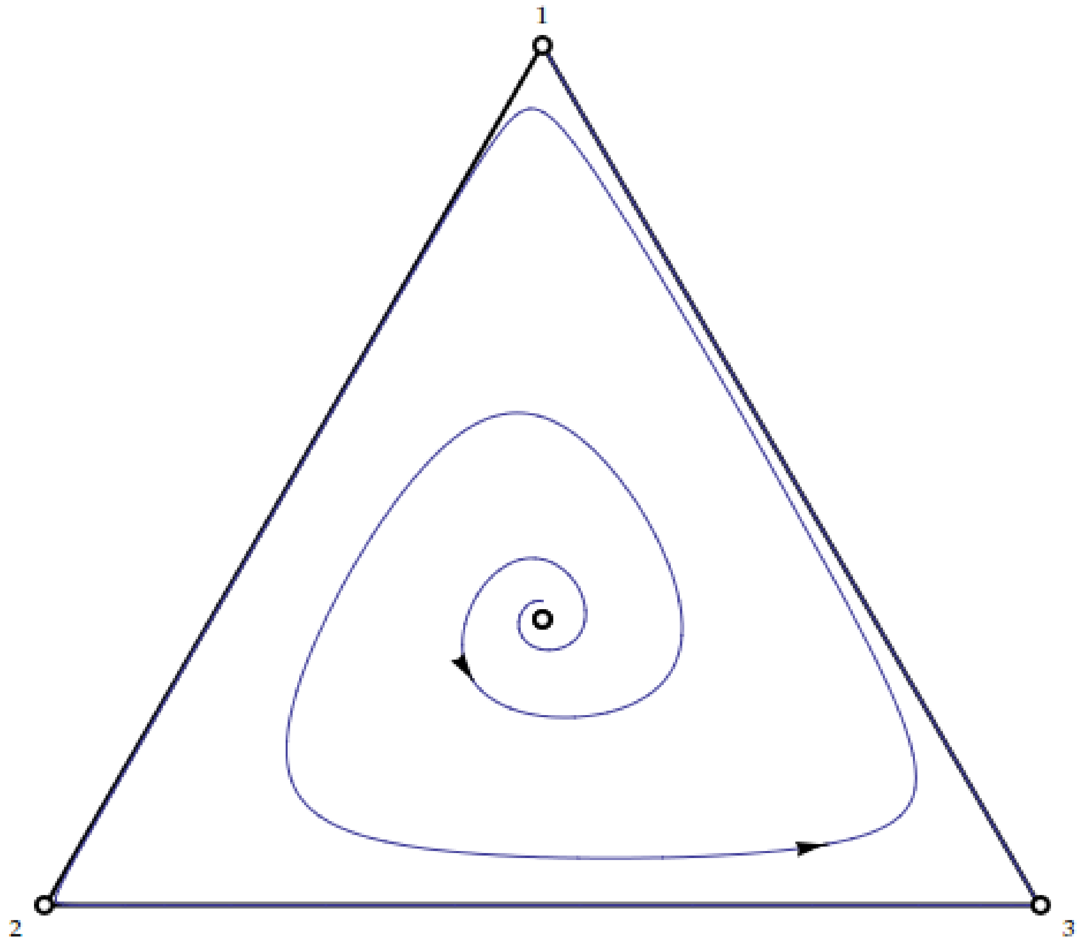


Example

	R	P	S
R	0	-2	1
P	1	0	-2
S	-2	1	0



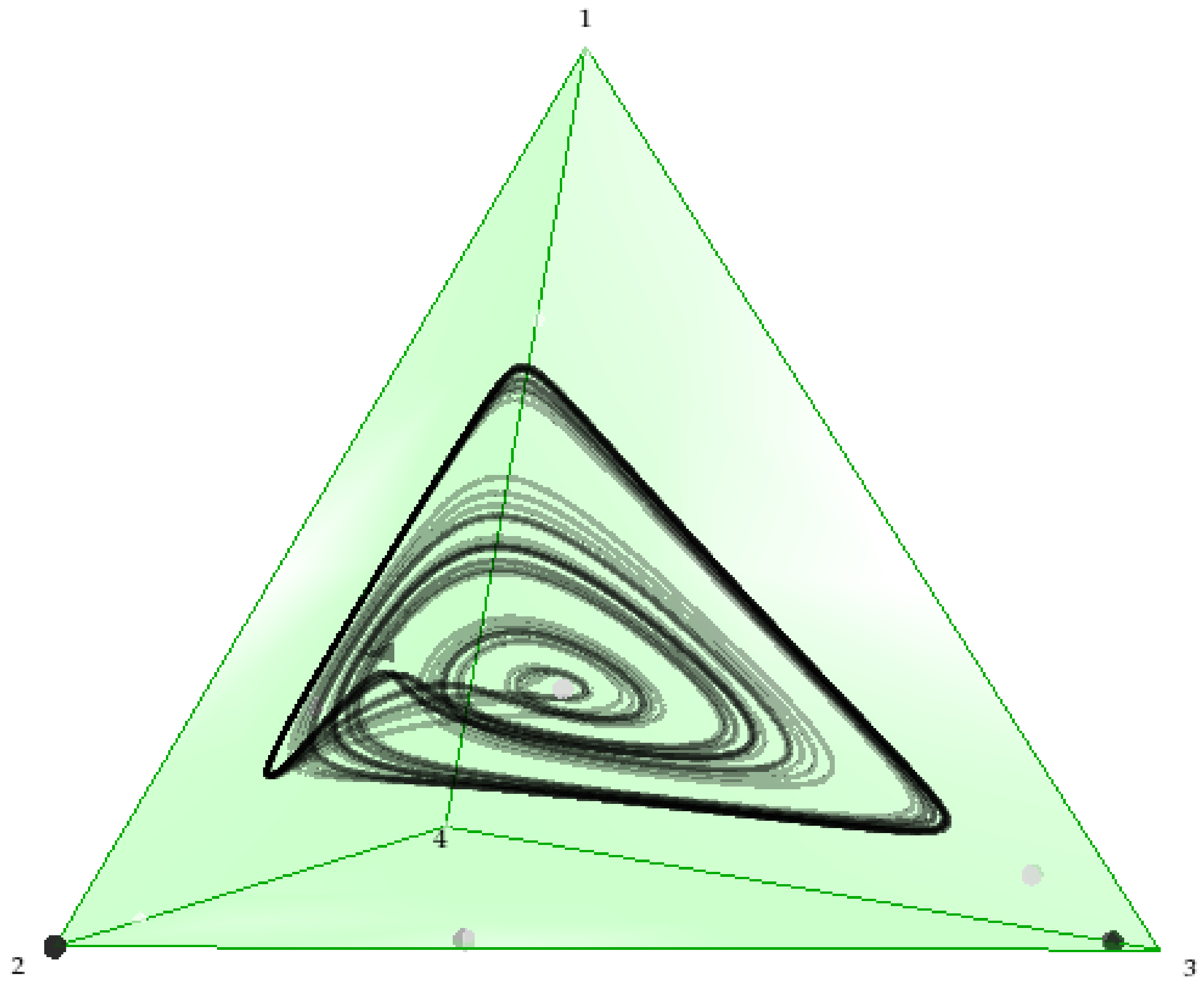
Avoid Nash



An odd game

	A	*B*	*C*	*D*
A	0	-12	0	22
B	20	0	0	-10
C	-21	-4	0	35
D	10	-2	2	0

Chaotic behavior



Not the real world

- Population is represented by a real number
  - Effectively infinite
- Each strategy gets exactly its expected payoff
  - Random matching
  - No stochasticity
- Time is continuous
  - No generations

Not the real world

- Reproduction/Imitation is entirely payoff based
  - No “drift”
- Strategies “breed true”
  - No mutation/experimentation
- Population is not shrinking
  - Extension is not an issue
- Reproduction is asexual
  - No recominbation

Finite populations

- All the same problems for Nash arise
- *Strictly dominated* strategies can invade

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Representing reasoning

- Logical language
  - Rationality
  - Features of the game
  - Knowledge of …
  - Knowledge of knowledge of …
- First order modal logic

Does this justify Nash?

The Nash equilibrium is the only outcome consistent with:

(a) Everyone knows everyone else's strategy

(b) Everyone is rational

Example

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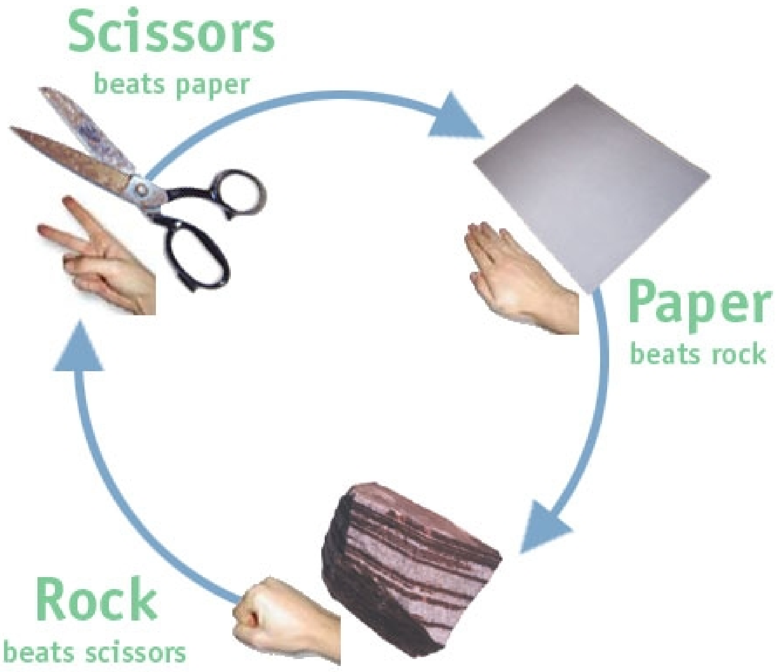

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(a) Everyone knows everyone else's strategy

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Common knowledge of rationality

- S_1: Everyone is rational
- S_2: Everyone knows S_1
- S_3: Everyone knows S_2

…

- S_n: Everyone knows S_{n-1}

…

S_∞: S_1 & S_2 & …

Does this justify Nash?

There exist non-Nash outcomes that are consistent with common knowledge of rationality and common knowledge of the game.

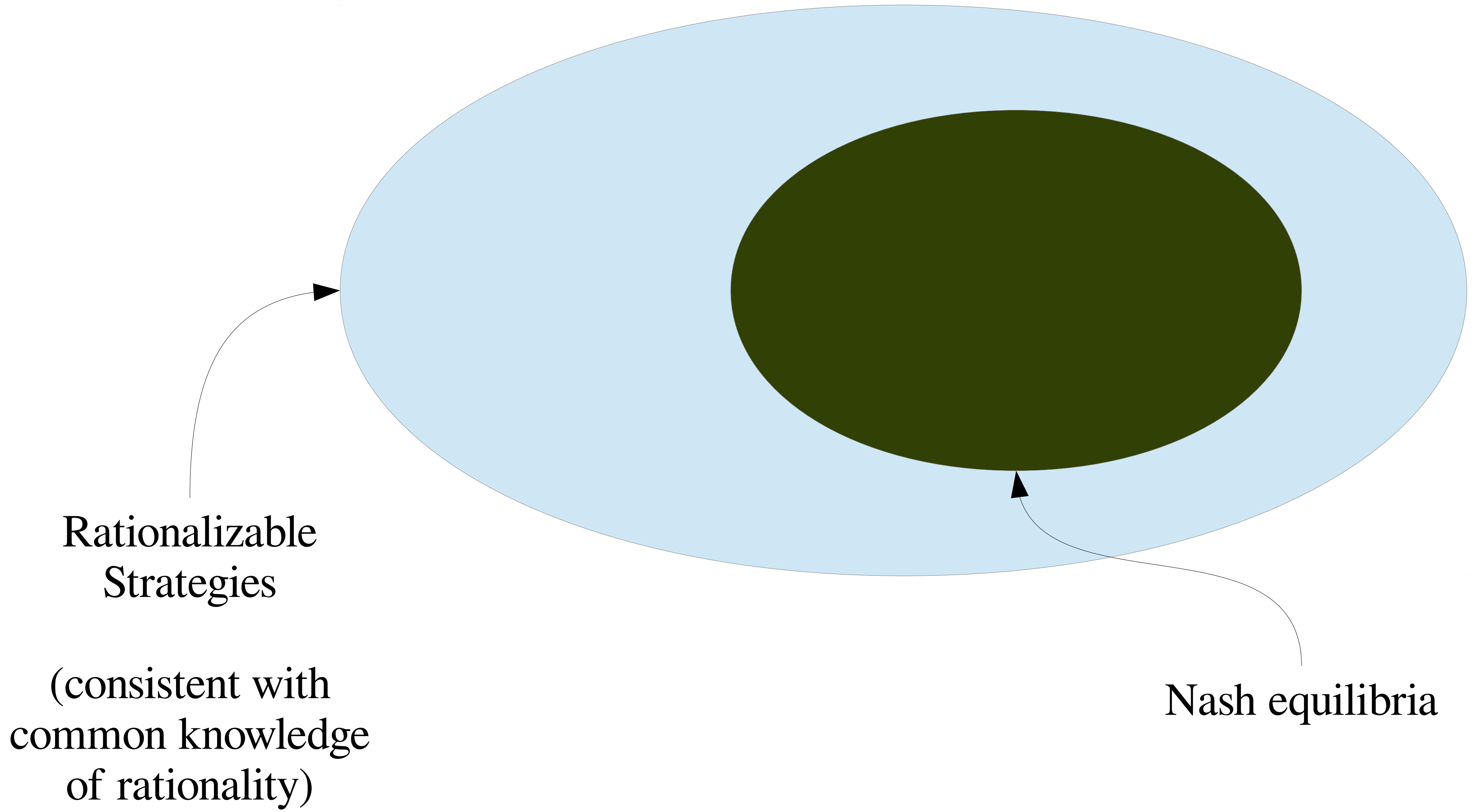
Example

	R	*P*	*S*
R	0	-1	1
P	1	0	-1
S	-1	1	0

- I think he will play rock
- Because I think he thinks I'm going to play scissors
- Because I think he thinks I think he's going to play paper

…

Strictly more general



Not the real world

- Knowledge is represented as absolute certainty
  - Do we ever have that?
- Common knowledge of rationality and the game
  - Do we know people that well?
- Infinitely long beliefs
  - S_∞ is an infinitely long sentence

Not the real world

- Logical omniscience
  - I know all the logical consequences of my beliefs
- Beliefs are all static
  - No process of reasoning
- Positive introspection
  - If I know *P*, I know that I know *P*

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Individuation of models

- Is game theory a model?
  - Or evolutionary game theory?
  - Or epistemic game theory?
- Is a game a model?
- Is a game with a dynamic a model?

Big question

Why all the trouble?

Options

- Galilean de-idealization
- Robustness testing
- Novel modeling programs

Options

- Galilean de-idealization
- Robustness testing
- Novel modeling programs

Galilean idealization

- We only idealize because we can't do better
- Better models are those that are more realistic
- Science is about constructing better and better models

Galilean de-idealization

- Traditional game theory leaves out change/beliefs
- Evolutionary/Epistemic GT reintroduces those
  - Are they “more” realistic?
  - Which is the correct de-idealization?

Two paths to reality

Evolutionary game theory

- Rationalizability is far too weak
- Nash is often too weak

Epistemic game theory

- Nash is too strong
- Variations on rationalizability are just right

What about traditional GT?

- Traditional game theory can only be used in those special cases endorsed by the foundational program(s)

- EGT replaces traditional game theory

- But scholars keep using traditional game theory

Options

- Galilean de-idealization
- Robustness testing
- Novel modeling programs

Robustness testing

- Some idealizations are okay, because they don't really make a difference
- Modeling tests whether their models are robust by embedding them in new models

EGT as robustness testing

- Are the conclusions of traditional game theory robust to variations in assumptions?
  - **NO!**
- Is that all these programs are doing?

Options

- Galilean de-idealization
- Robustness testing
- Novel modeling programs

Novel modeling program

- Not “foundational programs”
- Instead alternative models which should be tested independently
- What the inter-relation?

Levin's multiple models

- We come to understand the world by making many different idealizations
- EGTs are “multiple” models of the same phenomena
- Is there a general lesson?