Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

This homework is due before 5:00pm on December 16. Please leave the homework in Paul's box in Baker Hall 135. DO NOT LEAVE IT IN PROF. ZOLLMAN'S BOX!

Problem 1

Suppose that there are three candidates for election: A, B, and C. Suppose there are 9 voters distributed among these three profiles:

1. $A \succ B \succ C$ (*4 voters*)
2. $B \succ A \succ C$ (*2 voters*)
3. $B \succ C \succ A$ (*1 voter*)
4. $C \succ B \succ A$ (*2 voters*)

Part A

Who will win the election if voting is done by plurality vote (i.e. the person with the most first place votes wins) assuming that everyone votes honestly? Who will win if we do Borda count (again if everyone votes honestly)?

Part B

Suppose that the winner is determined by plurality vote. Should everyone vote honestly? I.e. is voting for her preferred candidate gogin to maximize a voter's utility assuming the other voters vote honestly? If the answer is no, who should switch and who should she vote for? Who would win? If she changed her vote, could anyone counteract her change by also changing his vote?

Part C

Suppose that the winner is determined by Borda count. Should everyone vote honestly? If no, who should switch and who should she vote for? Who would win? If she changed her vote, could anyone counteract her change by also changing his vote?

Problem 2

The Condorcet cycle is a collection of three people with the following preferences:

$$\begin{array}{ccc} a \succ & b & \succ c \\ b \succ & c & \succ a \\ c \succ & a & \succ b \end{array}$$

This preference profile can generate problems for a variety of voting rules. I said in class that this cannot arise with "single peaked preferences" in a single dimension. For instance, if the candidates can be described as "more liberal" or "more conservative" than one another, and if the voters choose an optimal level of liberal-ness and vote for the candidates based on how close the candidate is to the voter's ideal, then the Condorcet profile cannot arise. What if the candidates are ranked not in one dimension but two – like "socially liberal/conservative" and "fiscally liberal/conservative." Can the Condorcet profile arise in this circumstance? Either say why not, or give an illustration of it arising.

Problem 3

Consider the following scenario for a candidate. There is one dimension along which she can locate a position and it will be represented by $[0, 10]$. She must locate her position on a whole number $0, 1, \ldots, 10$. Voters are uniformly distributed over the space, for simplicity assume there is a voter for every real number on the line. Voters will vote for whichever candidate is closest to them on the line. Suppose there are only two candidates and one has already chosen his position, x. Calculate what the optimal position for the other candidate is assuming that she wants to maximize her votes.

Suppose now that you are the first candidate and you know that the second will maximize her expected number of votes. What should you do?

Graduate student problems (extra credit for undergrads

Problem 4

Recall the formal setup for Arrow. There is a finite set of alternatives P (which has at least 3 members). Let R be the set of all preference relations on P (let's use $\succsim$ here) For some arbitrary N, let $f : R^N \to R$.

A few bits of notation. Let $a \succsim_{x_i} b$ mean that in profile $x \in R^N$, individual i prefers a to b. Let $a \succsim_{f(x)} b$ denote the statement f applied to preference profile x prefers a to b

Consider the following conditions on f (these are the standard Arrow conditions):

- **Unrestricted domain**. $f(x)$ is defined for all $x \in R^N$
- **No dictator**. There is no i such that for all profiles x, $f(x) = x_i$
- **Positive Association** For all $x \in R^N$ and $a, b \in P$, if $a \succsim_{f(x)} b$, then for all profiles x', $a \succsim_{f(x')} b$ so long as x' satisfies these two conditions
  - For all i such that $a \succsim_{x_i} b$, $a \succsim_{x'_i} b$
  - There is some j such that $a \not\succsim_{x_j} b$ but $a \succsim_{x'_j} b$
- **Citizen Sovereignty** For all $a, b \in P$, there is some x such that $a \succsim_{f(x)} b$
- **I.I.A.** For all $x \in R^N$ and all $a, b \in P$, if $a \succsim_{f(x)} b$, then for all x', $aS_{f(x')}b$ so long as x' satisfies these two conditions
  - For all i, $a \succsim_{x_i} b$ if and only if $a \succsim_{x'_i} b$
  - For all i, $b \succsim_{x_i} a$ if and only if $b \succsim_{x'_i} a$

I said in class that in light of the other conditions Positive Association + Citizen Sovereignty is equivalent to Pareto.

- **Pareto** If for all i $a \succsim_{x_i} b$ then $a \succsim_{f(x)} b$

Prove this.