Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Give the most plausible illustration you can of someone who violates Sen's β.

Problem 2

Suppose a set X and a utility function $u(\cdot)$ over that set. We will work "backwards" and define a preference relation that fits this utility function in the following way. Let $x \succ y$ if and only if $u(x) > u(y) + 1$. Now let $x \sim y$ if and only if $x \not\succ y$ and $y \not\succ x$. Which of our 9 conditions are $\succ$ and $\sim$ guaranteed to satisfy? Which are they not guaranteed to satisfy? Explain why you came to the conclusions you did.

Problem 3

Part A

Using the 9 conditions we provided in class, prove that if $y \succ x$ and $y \sim z$ then $z \succ x$. (HINT: Can it be the case that $z \sim x$?)

Part B

Kreps (pg. 8) defines a property called negative transitivity. A preference relation $\succ$ is negatively transitive when its the case that:

- If $x \not\succ y$ and $y \not\succ z$, then $x \not\succ z$ ("$x \not\succ y$" means "not $x \succ y$")

Prove that negative transitivity holds for the preference relation as defined in class.

Graduate student problems (extra credit for undergrads)

Problem 4

Let's start with a choice set X and an arbitrary relation R. Let's define a "choice" function relative to R:

$$c(A, R) = \{x \in A | \text{ for all } y \in A, \text{ it is not the case that } yRx\}$$

Where A is any non-empty subset of X. If we think of R like preference, this says that the function $c(A, R)$ picks out only those options in A that are undominated by another option in A.

Part A

Prove that $c(\cdot, R)$ satisfies Sen's α. (Note this is somewhat surprising because we are making no assumptions about R at all, other than it's a binary relation)

Part B

Let's begin with a definition. A relation R is acyclic if for any set of x_i's, if $x_1 R x_2$, $x_2 R x_3$, $\ldots$, $x_{n-1} R x_n$ then $x_n \neq x_1$. Essentially a relation is acyclic if you cannot construct a circle with the relation. You should see immediately that our preference relation $\succ$ is acyclic. But there are other relations which are acyclic and don't satisfy the conditions we set out for $\succ$.

Give an example of an R that is acyclic, but where $c(\cdot, R)$ violates Sen's β.

Part C

Remember the non-emptiness condition for choice functions: if $A \neq \emptyset$ then $c(A, \succ) \neq \emptyset$.

Now take $\succ$ and $\sim$ to satisfy the nine conditions from class. Show that $c(\cdot, \succ)$ satisfies the non-emptiness condition, Sen's α, and Sen's β.