

Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Consider the following four Anscombe-Aumann style acts:

	S_1	S_2
f	0.75 – \$10 0.25 – \$0	0.75 – \$10 0.25 – \$0
g	\$ 0	\$ 0
h	0.8 – \$10 0.2 – \$0	\$ 0
i	\$ 0	0.8 – \$10 0.2 – \$0

Part A

Considering the addition operator, \oplus , from Anscombe-Aumann. Illustrate the following acts:

- $\frac{2}{3}f \oplus \frac{1}{3}g$
- $\frac{1}{2}g \oplus \frac{1}{2}i$
- $\frac{3}{4}f \oplus \frac{1}{4}h$

Which of them are constant acts?

Part B

What combination of only acts h and i would be equivalent to $\frac{1}{2}f \oplus \frac{1}{2}g$? Is there any combination of acts which is equivalent to $\frac{1}{2}f \oplus \frac{1}{2}i$? If so, what is it?

Problem 2

In class I gave the example of a violation of Anscombe-Aumann's axiom 5 involving your run-of-the-mill preferences over the following acts:

	<i>Rain</i>	<i>Sun</i>
<i>f</i>	Sunscreen	Sunscreen
<i>g</i>	Umbrella	Sunscreen
<i>h</i>	Sunscreen	Umbrella

Part A Reproduce the violation of axiom 5 with these three acts. Show why this violates axiom 5.

Part B I said in class that this problem could be solved by replacing the outcomes “umbrella” and “sunscreen” with different, more subjective, outcomes like “being sunburned” or “being wet”. Illustrate how this would solve this problem.

Part C Come up with another example of state-dependent utilities (not used in class or in the book). Show how your example violates axiom 5. Can it be solved using this same trick – by redescribing the outcomes in a different way?

Problem 3

Suppose we begin with a prize set that contains three soccer jerseys $Z = \{ \text{Barcelona, Manchester United, FC Bayern} \}$. Carlos assigns the following utilities to the three jerseys,

$$\begin{aligned} u(\text{Barcelona}) &= 7 \\ u(\text{Manchester United}) &= 3 \\ u(\text{FC Bayern}) &= 0 \end{aligned}$$

Carlos also has a probability distribution over a set of states of the world, S . If you give Carlos two lotteries or acts over different jerseys he will dutifully calculate the expected utilities of the lotteries or act and prefer the lottery or act with higher expected utility. But, there is one caveat: Carlos' calculator rounds off to the nearest two decimal places. So if the expected utilities of two lotteries or acts differ but round off to the same number, Carlos will regard them as equivalent (and will therefore be indifferent).

Will Carlos violate any of the axioms of Anscombe-Aumann? If so, which ones? How?

Graduate student problems (extra credit for undergrads)

Problem 4

Suppose a preference relation, \succsim over acts which satisfy the Anscombe-Aumann axioms. I find a utility function, $u(\cdot)$ that represents this preference relation over constant acts. (Recall a constant act is an act that gives the same objective-probability gamble in each state.) I want to know if I can always be guaranteed to find a probability function $p(\cdot)$ such that p and u represent \succsim .

Either (a) prove that for all u that represent \succsim over constant acts, there exists a probability function $p(\cdot)$ such that p and u together represent \succsim over all acts or (b) give a counterexample to this claim.