Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Part A

Let's start with a prize set that contains three beers $Z = \{$ Flying dog, Penn, Weyerbacher $\}$. Suppose that Jake has the following utility function:

$$\begin{aligned} u(\text{Flying dog}) &= 1 \\ u(\text{Penn}) &= 0.5 \\ u(\text{Weyerbacher}) &= 0 \end{aligned}$$

Suppose that Jake decides between two gambles in the following way. If a gamble p is just a sure thing gamble (one prize has probability 1), then he assigns that gamble the utility of it's outcome. If a gamble is not a sure thing, he assigns that gamble it's expected utility *minus 0.1*. So for instance, the gamble that gives a 50% chance of Flying dog and a 50% chance of Weyerbacher is valued at $0.5 - 0.1 = 0.4$.

Jake now constructs a preference relation over all gambles based on this utility function (he prefers those gambles which are assigned higher utility by him).

- Will Jake violate Axiom 1 of von Neumann/Morgenstern? If so, how? If not, why do you think not?
- Will Jake violate Axiom 2 (independence) of von Neumann/Morgenstern? If so, how? If not, why do you think not?
- Will Jake violate Axiom 3 (continuity) of von Neumann/Morgenstern? If so, how? If not, why do you think not?

Part B

Answer the same three questions again, but now suppose that the penalty for gambles is higher, now it's 0.5. So the gamble 50% Flying dog, 50% Weyerbacher is worth $0.5 - 0.5 = 0.0$.

Problem 2

Consider Shannon who uses maximax to choose between two different gambles. She starts with a preference relation $\succ$ over the prizes in Z, and then compares two different lotteries using this preference relation. She looks at the best prize (according to $\succ$) that has non-zero probability in each lottery and compares them. If one lottery, p, gives a better best prize than another, q, she chooses p. If they are tied she looks at the second best prize (according to $\succ$) – she uses lexicographic maximax.

Does Julie violate any of the axioms of von Neumann/Morgenstern? If so, which ones? Illustrate any violations that you might find.

Problem 3

Recall the lexicographic preference ordering we discussed in class. I can choose two things: a proportion of the remaining macaroni salad ($x \in [0, 1]$) and a proportion of the remaining mashed potatoes ($y \in [0, 1]$). My choice set is made up of pairs (x, y) that represent the food I'm given. Suppose I choose first on the basis of macaroni salad, I want more rather than less. So $(x, y) \succ (w, z)$ if $x > w$. If the two options have the same amount of macaroni salad, only then do I compare the amount of mashed potatoes (again I want more). So $(x, y) \succ (w, z)$ if $x = w$ and $y > z$.

We can extend this to gambles as well. First I compare to gambles on the expected amount of macaroni salad (choosing the gamble which gives a higher expected amount of macaroni salad). Then if they are tied I choose on the basis of expected amount of mashed potatoes.

This preference relation violates at least one of the von Neumann/Morgenstern axioms. Which one(s)? Show why it violates each one.

Graduate student problems (extra credit for undergrads)

Problem 4

Recall the statement I proved in class

Definition 1 (Fact 2)**.** *For all p, q, r if $p \succ q \succ r$ then there exists an $a \in [0, 1]$ such that $ap \oplus (1 - a)r \sim q$.*

I said that you could replace axiom 3 with this statement. Prove me right. That is show that Axiom 1, Axiom 2, and Fact 2 entails Axiom 3.

Problem 5

In problem 3 I gave you a preference relation which violates von Neumann/Morgenstern. Can you come up with an alternative axiomatization which preserves the general spirit of von Neumann/Morgenstern, but allows for preference relations of this type? We know that you cannot capture this preference relation with a single valued utility function, but can you come up with some other type of representation theorem? Give it your best shot.