Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Choose one of the offered interpretations of probability and write 2-4 paragraphs (1-2 double spaced pages) criticizing that view as the unique correct view of probability. To remind you, the offered interpretations were: propensity, objective relative frequency, hypothetical relative frequency, or subjective confidence (aka credence or subjective Bayesianism).

Problem 2

Part A

Someone has been killed by a machete, and you are the investigating detective. You have found a suspect, Julie. You initially estimate that the probability she committed the murder is 1/100. You plan to search her house, and before you do you estimate the following probabilities. If Julie committed the murder, the probability you will find a machete in her house is 3/4 (she might have disposed of the machete). If Julie didn't commit the murder, the probability you will find a machete is 1/3 (some innocent people own machetes). You search Julie's house and find a machete. What's the probability she committed the murder?

Part B

Now that you've found the machete, you decide to test Julie for blood residue. You estimate that if she committed the murder the probability of blood residue will be 2/3. If she didn't commit the murder, the probability of finding blood residue is 1/2 (after all people who own machetes might cut themselves). You test Julie and find blood residue. What's the probability she committed the murder now?

Problem 3

Recall the value of information problem in class. I had in my pocket two coins, one was fair ($P(\text{Heads}) = 0.5$) one was biased ($P(\text{Heads}) = 0.25$). I offered you the following gamble, if the coin came up heads I would pay you \$3, but if it came up tails you would pay me \$2.

I'm now going to give you six options. You could take the gamble immediately, you could refuse the gamble immediately, or you could choose to pay some amount a to observe a flip of the coin. If you chose to observe the flip, you saw whether the coin came up heads or tails and then you can take the gamble or refuse it – but in either case you had to pay me a for the information. (Do you see what the six options are?)

For what values of a does the action "Pay a, and then take the gamble if the coin comes up heads and refuse if tails" maximize expected utility compared to the others? What if instead of observing a test flip, I told you for sure whether the coin was fair or biased? What is the most you would be willing to pay then?

Problem 4

The St. Petersburg lottery. Suppose you meet Jake who can (credibly) offer you the following gamble. Jake offers to flip a fair coin until it comes up head for the first time. He will then count the total number of flips and pay you $\$2^n$ where n is the total number of times he flipped the coin. If the coin came up heads on the first flip, you get \$2, if it came up *tails-heads* you get \$4, if it came up *tails-tails-heads* you get \$8, etc.

First, how much would you be willing to pay Jake for the opportunity to play this game? (This is not a math question, just think about it to yourself for a minute and tell me what you would do.)

Second, suppose that Shannon's utility function for money is the identity function (i.e. the utility of \$1 is 1, the utility of \$2 is 2, etc.). What is the expected utility of this gamble for Shannon? How much do you think Shannon should be willing to pay if she is an expected utility maximizer?

Do you and Shannon differ? If so, why do you prefer your answer?

Graduate student problems (extra credit for undergrads)

Problem 5

Suppose a decision problem with n acts, $A_1, \ldots, A_n$, m states, $S_1, \ldots, S_m$, and you have a probability distribution over states given by $p(\cdot)$. The value of this decision problem, V, is just the expected utility of the action with highest expected utility.

Now suppose a partition on the states $B = \{b_1, \cdots, b_r\}$ (where $r \leq m$ of course) and that I will tell you what partition we are in before you are required to choose an action. Prove that before I inform you of the partition, you now think this new decision problem has a value of at least V. Show that there are some cases where it might be greater than V.

This shows that the value of information is always greater than or equal to zero.

(HINTS: (1) Consistency requires that the probability you think I will say we are in b_x is given by $p(b_x)$. (2) The expectation of an act can be represented as the weighted sum of the expectation of that act in each partition.)

Problem 6

Suppose we have a choice set X which is composed of all pairs of real numbers between 0 and 1 (inclusive), $X = [0, 1] \times [0, 1]$. I illustrated a preference relation which cannot be captured by any utility function called a *lexicographic* preference relation.

- $(x_1, y_1) \succ (x_2, y_2)$ if $x_1 > x_2$
- $(x_1, y_1) \succ (x_2, y_2)$ if $x_1 = x_2$ and $y_1 > y_2$, and
- $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 = x_2$ and $y_1 = y_2$

Show that this violates *continuity of* $\succ$ defined here,

Definition 1. *A relation $\succ$ on X is continuous if for all sequences $\{x_1, x_2, \ldots, x_n\}$ from X with limit x two things hold:*

- *if $x \succ y$ for some $y \in X$, then for all large n, $x_n \succ y$, and*
- *if $y \succ x$ for some $y \in X$, then for all large n, $y \succ x_n$.*