Problem 1

We talked about using Bayes rule to modify your probabilities on the basis of new evidence. The idea was that if I gave you some new information today your probabilities today should be your probabilities from yesterday conditioned on the new information. Let P_T be your probability function today and P_Y be your probability function yesterday and let I be the information I gave you. Formally we represent this "principle of conditionalization" by saying:

$$P_T(x) = P_Y(x|I)$$

In this problem I want you to show that if you update in this way it doesn't matter in what order information arrives. Specifically suppose you get two pieces of information (I_1 and I_2) and update on them. You could update in one order:

$$\begin{aligned} P_2(x) &= P(x|I_1) \\ P_3(x) &= P_2(x|I_2) \end{aligned}$$

Or you could update the other way:

$$\begin{aligned} P_2'(x) &= P(x|I_2) \\ P_3'(x) &= P_2'(x|I_1) \end{aligned}$$

Show that regardless of the probability function, $P_3(x) = P_3'(x)$. You can use either the set theoretic or the propositional probability calculus we discussed in class, just be consistent.

Problem 2

von Neumann and Morgenstern show a method by which a utility function can be constructed from preferences over betting. This method is often used by financial advisers and others to determine a person's utility function over money. They begin by assigning $u(\$0) = 0$. They then ask their subject to compare gambles to sure things until they find indifference points.

I want you to do this with a friend. Go through \$10 increments from \$10 to \$100 and find gambles which they regard as equally valuable as getting that dollar amount for sure. (One way to do this is ask what amount of money they would be willing to pay in order to get the gamble. Alternatively, you could ask them what price they would demand to offer the gamble to someone else. These might not be the same, though, so you should be consistent throughout your "experiment".)

Once you've done this construct a utility function which represents their preferences, and justifies their choices on the basis of maximizing expected utility. It doesn't need to be mathematically well defined for all possible dollar amounts you can just say $u(\$10) = 7$, $u(\$20) = 10$, etc. How does your friend's utility function compare to someone who values each dollar as much as every other one (e.g., $u(\$x) = x$)?

Problem 3

Kreps defines conditional preference for Savage's theory in Definition (9.4). He then goes on to assert (Lemma 9.5) that this conditional preference relation satisfies axioms (9.1) and (9.3). Prove this.

Problem 4

Consider a condition that I will call Set Independence:

Definition 1 (SI). *Suppose a set of alternatives S and two group profiles P_1 an P_2 that both rank members of S in the same order. Then $f(P_1)$ and $f(P_2)$ both rank the members of S in that order.*

Show that (SI) is equivalent to Arrow's independence of irrelevant alternatives condition. That is, show that any function which satisfies independence of irrelevant alternatives will satisfy (SI) and that any function which satisfies (SI) will satisfy independence of irrelevant alternatives. (HINT: The first part is the hard part, and you will need to use mathematical induction.)

Graduate student problems (extra credit for undergrads)

Problem 5

One can construct pathological cases for majority rule with three people by having preferences of the following sort:

$$a \succ b \succ c$$
$$b \succ c \succ a$$
$$c \succ a \succ b$$

Black showed that in one dimension if people have single peaked preferences this profile cannot arise. It can arise in two dimensions however. Construct an example that generates this profile by choosing three candidates and locating their positions in $\mathbb{R}^2$. Have three voters choose "ideal" points in that same space and then choose candidates based on their Euclidean distance from their ideal points (obviously closer is better). Show that one can place candidates and ideal points such that the pathological profile is generated.

Problem 6

Consider the following scenario for a candidate. There is one dimension along which she can locate a position and it will be represented by $[0, 10]$. She must locate her position on a whole number $0, 1, \ldots, 10$. Voters are uniformly distributed over the space, for simplicity assume there is a voter for every real number on the line. Voters will vote for whichever candidate is closest to them on the line. Suppose there are only two candidates and one has already chosen his position, x. Calculate what the optimal position for the other candidate is assuming that she wants to maximize her votes.

Suppose now that you are the first candidate and you know that the second will maximize her expected number of votes. What should you do?