Written answers are acceptable so long as they are legable. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

On homework 1, I gave you the following game: There is a deck of cards that contains two types of cards "High" and "Low". You and "the dealer" are each dealt one card from this deck. You observe the card you have been dealt (but not the card the dealer has received), and are given the option of "surrendering". If you surrender, you give the dealer \$50. If you do not surrender, you compare your card to the dealer's. If you have the high card and the dealer has the low, the dealer pays you \$100. If you have the low and the dealer has the high you pay the dealer \$100. If you both have the same card, nothing happens

Suppose that the probability of a high card is p, determine what strategy would maximize expected utility (it might be different for different values of p).

Problem 2

Recall the coin flipping problem from class and from homework 2. There you could pay an amount a in order to observe a "test flip" of the coin, and we calculated how much you would be willing to pay in order to observe that flip. (More formally, we calculated what the highest value of a such that "Pay, then take if heads and refuse if tails" still had the highest expected utility of all options.) Suppose that instead of observing a test flip, I will just tell you whether the coin is fair or not. What is the highest amount that you should be willing to pay to secure that information?

Problem 3

Suppose a normal form decision problems with two actions A_1 and A_2 and n states, $S_1, \ldots, S_n$. Suppose I add another action A_3 such that in every state A_3's outcome is the sum of the outcomes of A_1 and A_2 in that state. Show that regardless of the probability distribution, number of states, or actual payoffs the expected utility of action A_3, represented as $EU(A_3)$, equals $EU(A_1) + EU(A_2)$.

Problem 4

In class, and in the book, I only showed how to construct a Dutch book when a and b are mutually exclusive and $P(a \text{ or } b) < P(a) + P(b)$. Show the other case. Suppose that a and b are mutually exclusive, but Savi assigns $P(a \text{ or } b) > P(a) + P(b)$. Show how to construct a Dutch book against him.

Graduate student problems (extra credit for undergrads)

Problem 5

Recall the "distance argument" for consistency that I discussed in class. You make an estimate p on an event A happening and q on it not happening. Suppose that I charge you for your distance from the truth, that is, if A happens I charge you $(1-p)^2 + (0-q)^2$ utiles and if A doesn't happen I charge you $(0-p)^2 + (1-q)^2$ utiles. Show that (in this limited case) for any incoherent estimate (where $p \neq (1-q)$) there is a consistent estimate that strictly dominates it. You may for the purposes of this problem assume that all estimates are in $[0, 1]$, even if inconsistent.

Problem 6

Here is a general version of an earlier problem. Suppose a decision problem with n acts, $A_1, \ldots, A_n$, m states, $S_1, \ldots, S_m$, and you have a probability distribution over states given by $p(\cdot)$. The value of this decision problem, V, is just the expected utility of the action with highest expected utility.

Now suppose a partition on the states $A = \{a_1, \cdots, a_r\}$ (where $r \leq m$ of course) and that I will tell you what partition we are in before you are required to choose an action. Prove that before I inform you of the partition, you now think this new decision problem has a value of at least V. Show that there are some cases where it might be greater than V.

This shows that the value of information is always greater than or equal to zero.

(HINTS: (1) Consistency requires that the probability you think I will say we are in a_x is given by $p(a_x)$. (2) The expection of an act can be represented as the weighted sum of the expectation of that act in each partition.)