

Written answers are acceptable so long as they are legable. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Resnik uses the following version of the continuity axiom:

Axiom 1 (Continuity). *For any lotteries x , y , and z such that $x \succ y \succ z$, there is a real number $a \in [0, 1]$ such that $y \sim L(a, x, z)$.*

There is another version of this axiom. We'll call it "Weak Continuity"

Axiom 2 (Weak Continuity). *For any lotteries x , y , and z such that $x \succ y \succ z$, there are real numbers $a, b \in [0, 1]$ such that $L(a, x, z) \succ y \succ L(b, x, z)$.*

Prove Weak Continuity using the axioms of Von Neumann/Morgenstern presented in Resnik.

Problem 2

Suppose that I make all my decisions on the basis of minimax. That is, given the choice between two actions or lotteries I will choose the one whose worst case outcome is best. Which axiom(s) of Von Neumann/Morgenstern do I violate? Show how.

Problem 3

Suppose I make my decisions on the basis of minimax like before. Which axiom(s) of Savage do I violate? Show how. What if I use lexicographic minimax? Which ones will I violate then? Show how.

Problem 4

For each of Savage's axioms 9.3, 9.7, and 9.8 give an example of a preference relation that violates it. (Obviously you don't need to consider all possible acts, just give a few acts and a preference relation that violates it.)

Graduate student problems (extra credit for undergrads)

Problem 5

Suppose S is a set of *three* objects and that A is the set of all subset of S . Let \succsim be a qualitative probability defined on A . Prove that there is a *probability function* $p : A \rightarrow [0, 1]$ such that: $x \succsim y$ if and only if $p(x) \geq p(y)$.

For lots of extra credit show that this is also true when S has four objects.

Problem 6

Consider Savages theory. Suppose that $f \succ g$ given A and that $f \succ g$ given B and that $A \cap B = \emptyset$. Prove (without using the representation theorem) that $f \succ g$ given $A \cup B$. Would this still hold if $A \cap B \neq \emptyset$?