Written answers are acceptable so long as they are legable. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Choose one of the offered interpretations of probability and write 2-4 paragraphs (1-2 double spaced pages) criticizing that view as the unique correct view of probability. To remind you, the offered interpretations were: objective chance, objective relative frequency, hypothetical relative frequency, or subjective confidence.

Problem 2

Construct a normal form decision with four options such that maximax, minimax, minimax regret, and maximize expected utility each suggest a different option. Illustrate why each decision rule chooses each action (be sure and provide a probability distribution over the states for the maximize expected utility decision rule).

Problem 3

Consider the following four decision rules, maximax, minimax, lexicographic minimax, and maximize expected utility. Will each of them ever select a strictly dominated strategy? What about a weakly dominated strategy? For each possibility either give an illustration where they will choose a dominated strategy or prove that they will never make that choice.

(HINT: If you think they will not select a dominated strategy, suppose that you have two actions A_1 and A_2 and suppose that A_1 dominates A_2. Show that regardless of the number of states and the magnitude of the payoff the decision rule will select A_1.)

Problem 4

Recall the value of information problem in class. I had in my pocket two coins, one was fair ($P(\text{Heads}) = 0.5$) one was biased ($P(\text{Heads}) = 0.25$). I offered you the following gamble, if the coin came up heads I would pay you \$3, but if it came up tails you would pay me \$2. I gave you three options. You could take the gamble immediately, you could refuse the gamble immediately, or you could choose to pay some amount a to observe a flip of the coin. If you chose to observe the flip, you then had the opportunity to take the gamble or refuse it – but in either case you had to pay me a for the information.

I said in class that there were six total strategies, but two of them were dominated. Prove that these two are strictly dominated by another strategy (you can either write a normal form representation of the game or divide the circumstance into cases to demonstrate dominance).

Graduate Student Problems (extra credit for undergrads)

Problem 5

A Bandit problem Suppose you are confronted by two different slot machines. On each pull it either gives you a win of \$1 or you lose \$1. You are told that one slot machine will give a win with probability 0.5. The other slot machine is either a good slot machine, which gives wins with probability $0.5 + \alpha$, or a bad slot machine, which gives wins with probability $0.5 - \alpha$ (where $0 < \alpha < 0.5$) – it is equally likely to be either. You are obliged to play twice – first you choose a machine, play it, and observe the outcome, then you choose again and play again. Compare two strategies. On one you play the known machine on both pulls. On the other, you pull the unknown machine and if it wins you play it again, otherwise you switch to the known machine. Show which of these strategies maximize expected utility (it might depend on α).

(NOTE: This problem involves a lot of algebra. You may use a computer algebra program like Mathematica to help you, but you need to illustrate the high points of the solution.)

Problem 6

The rich get richer Suppose Bill who has an unlimited betting amount meets a bookie who also has an unlimited betting amount. The bookie promises to offer Bill a gamble which has some probability $p > 0$ of resulting in a win for Bill. The bookie will continue offering Bill the gamble as long as Bill is willing to play, and he guarantees that each play is independent of the previous plays. Bill can choose how much to gamble at each stage. A win results in the bookie paying Bill the amount he bet, otherwise Bill pays the bookie that amount. Bill reasons that he can guarantee a win by adopting the following strategy. Bill will start with an amount $\$a$ that he would like to win. He bets $\$a$ on the first play and if he wins, he quits and walks away. If he loses he will bet $\$2a$ on the next game. If he wins he quits, and if he loses he plays again and bets $\$4a$. He will continue to double until he wins, but as soon as he wins he quits. Show that this strategy has an expected utility of $\$a$ regardless of p. Once you've done this, say why shouldn't I go to Vegas with this strategy.