Written answers are acceptable so long as they are legable. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Consider the following simple gambling game. There is a deck of cards that contains two types of cards "High" and "Low". You and "the dealer" are each dealt one card from this deck. You observe the card you have been dealt (but not the card the dealer has received), and are given the option of "surrendering". If you surrender, you give the dealer \$50. If you do not surrender, you compare your card to the dealer's. If you have the high card and the dealer has the low, the dealer pays you \$100. If you have the low and the dealer has the high you pay the dealer \$100. If you both have the same card, nothing happens. Represent this decision problem in both extensive and normal forms.

Just for fun, think about what you would do if you played this game. Does the right strategy depend on what the probability of a High card is? What would change if a High card was more likely or less likely?

Problem 2

In class the relations $\succ$ and $\sim$ were defined as having the following properties:

- $\succ$ is irreflexive, asymmetric, and transitive
- $\sim$ is reflexive, symmetric, and transitive
- If $x \succ y$ then $x \nsim y$ ("$x \nsim y$" means "not $x \sim y$")
- For all x and y either $x \succ y$ or $y \succ x$ or $x \sim y$
- If $x \succ y$ and $y \sim z$ then $x \succ z$

In class I said that if $\succ$ is a preference relation which conforms to the properties listed above then it cannot be the case that there are x, y, and z such that, $x \succ y$, $y \succ z$ and $z \succ x$. Prove that this is inconsistent with some of the above constraints.

Problem 3

Using the conditions from problem 2, prove that if $y \succ x$ and $y \sim z$ then $z \succ x$. (HINT: Can it be the case that $z \sim x$?)

One can also use proofs like this to show that the last requirement, joint transitivity, is already required by the earlier conditions. For extra-credit prove this.

Problem 4

Kreps (pg. 8) defines a property called negative transitivity. A the preference relation $\succ$ is negatively transitive when its the case that:

- If $x \not\succ y$ and $y \not\succ z$, then $x \not\succ z$ ("$x \not\succ y$" means "not $x \succ y$")

Prove that negative transitivity holds for the preference relation as defined in class (the conditions listed in problem 2).

(FIRST HINT: Consider all the possible cases for how x, y, and z might be related. SECOND HINT: For one of these cases it might help to use the fact you proved in one of the previous problems in this homework.)

Problem 5

Suppose that $u(x)$ is an ordinal utility function. We demonstrated in class that there are many functions $f(x)$ such that if $u(x)$ represents a preference relation then so does $f(u(x))$. We called these "transformations". Suppose we take a utility function $u(x)$ which represents some preference relation $\succ$ and suppose that we take $f(x) = 2x$, can we be guaranteed that $f(u(x))$ represents $\succ$? If not, give an example where it fails. What about $f(x) = 2x^2$? Again, if not give an example where it fails.

Graduate student problems (extra credit for undergrads)

Problem 6

Prove Proposition (2.12) from Kreps (pg. 14).

Problem 7

Suppose a set X and a utility function $u(\cdot)$ over that set. We will work "backwards" and define a preference relation that fits this utility function in the following way. Let $x \succ y$ if and only if $u(x) > u(y) + 1$. Now let $x \sim y$ if and only if $x \not\succ y$ and $y \not\succ x$. Which of our conditions (listed in problem 2) are $\succ$ and $\sim$ guaranteed to satisfy? Which are they not guaranteed to satisfy? Give proofs or counter-examples for each.