I. Shannon's model of communication

A. Information source -¿ Transmitter -¿ (noise source) -¿ Receiver -¿ Destination

B. Shannon imagined this whole system was designed by a particular person to acheive a particular use

C. But what if the Information source and transmitter are one person and the receiver and destination are another. (And maybe the noise source a third person?)

D. Now we have a more complicated question. Not only are we interested in the information capacity of the channel, but we are interested in why the transmitter is willing to transmit information to the receiver

E. If we expand this picture to include more than one transmitter and receiver, we can then ask, how do they arrange themselves in such a way to make the best transmission system?

1. We'll look at the first question this morning

2. The second we'll save for this afternoon

II. Common Interest Signaling game

A. Let's start with a very simple version of Shannon's situation.

1. The world is the information source and decides on one of two options

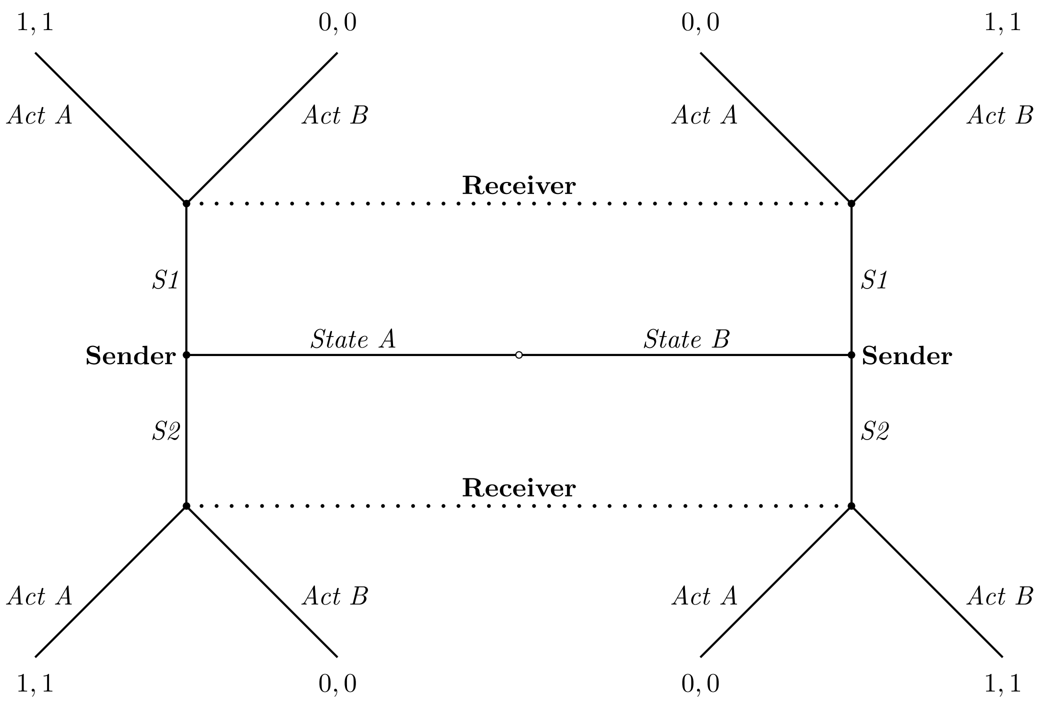
2. The transmitter is a person, called the "sender" who observes the world and then transmits one of two potential signals to the receiver

3. Assume there is no noise source, the receiver gets the message sent

4. The receiver then observes the signal and chooses one of two actions. Which one is better depends on the state of the world.

5. In the beginning assume that the sender and the receiver have common interest. They both want the same thing in each state of the world.

B. Draw the extensive form (explaining what information sets are)



C. Graph of strategies

1. The sender strategy is a function from states of the world to signals. In this case σ : { *State A, State B* } $\rightarrow$ { *S1, S2* }.

2. The receiver strategy is also a function, this time from signals to acts. ρ : { *S1, S2* } $\rightarrow$ { *Act A, Act B* }.

D. Nash of this game

1. Recall what a Nash equilibrium is. Each player is doing the best she can *given the choice of the other.*

2. Signaling systems

3. Pooling (do the best action or anything when equiprobable)

E. Significance of Nash

1. Perfect information tramission is possible

2. But not guarenteed

3. One needs to approach this problem using more sophisticated tools that allow one to predict under what circumstances one equilibrium arises rather than another

i. this is EGT. It seems to suggestion some information transfer is inevitable

ii. But as the game becomes more complex, not always perfect

III. Homework

A. 3x3x3 Lewis signaling game

1. Pooling equilibria

2. Partial Pooling equilibria

3. Signaling systems

B. Differential cost

1. If $p < 0.5$, there is an equilibrium where neither type sends the expensive signal and the firm doesn't hire regardless of signal

2. If $p > 0.5$, there is an equilibrium where neither type sends the expensive signal and the firm hires when it receives the cheap signal

3. If $c_l > 1 > c_h$, then there is an equilibrium where the high type sends the expensive signal and the low type does not, and the firm only hires if it receives the expensive signal

C. Differential benefit

1. If $p < 0.5$, there is an equilibrium where neither type sends the expensive signal and the firm doesn't hire regardless of signal

2. If $p > 0.5$, there is an equilibrium where neither type sends the expensive signal and the firm hires when it receives the cheap signal

3. If $a > c > b$, then there is an equilibrium where the high type sends the expensive signal and the low type does not, and the firm only hires if it receives the expensive signal

IV. Crawford and Sobel model

A. Motivation

1. So far we've consider two cases

i. Both players have identical interests

ii. There are two cases, (1) identical interest and (2) divergent interests

2. What about similar, but not perfectly matching interests

3. One really can't do that with a few number of actions. Instead, we'll move to a richer structure

B. Setup

1. Suppose the state of the world is a real number drawn from $[0, 1]$

i. For the moment we'll presume this is drawn from a uniform distribution, but it's possible to generalize

2. The sender observes the state of the world and sends a signal also from $[0, 1]$

3. The receiver observes the signal and chooses a number from $\mathbb{R}$

C. Common interest case

1. Suppose that both the sender and the receiver want the receiver to say a number that equals the state of the world

i. We'll suppose that they are charged based on squared error loss

ii. If the state of the world is s and the receiver guesses x, both players receive $-(s-x)^2$

2. Many equilibria

i. Signaling system equilibrium, the sender adopts a 1-1 function from states to signals, the receiver adopts the inverse of that function. Both players do perfectly

ii. Another equilibrium. The sender send 0 regardless of state, and the receiver guesses 0.5 regardless

iii. Another equilibrium. The sender sends 0 if $s \leq 0.5$ and 1 otherwise. The receiver guesses 0.25 if $s = 0$, 0.75 if $s = 1$, and guesses 1 otherwise.

iv. Many more: there exists an equilibrium that partitions the state space into n partitions for any $n \in \mathbb{N}$.

D. Divergent interest case

1. Suppose that they disagree about what number they want the receiver to guess

i. The receiver wants to guess s, but the sender wants the receiver to guess $s - 0.1$

ii. Again squared error loss, receiver: $-(s-x)^2$, sender $-(s-0.1-x)^2$

2. Fewer equilibria

i. Total pooling: sender sends 0 regardless of state, receiver guessesd 0.5 regardless of signal

ii. 2-partion case:

a. Suppose that the sender chooses some number, say a_1 and sends 0 if $s \leq a_1$ and 1 otherwise

b. Given that this is ther senders strategy, the receiver should guess $a_1/2$ if 0, and $a_1 + 1/2$ otherwise

c. Given this will be the receiver's strategy, how sould the sender respond?

d. If the sender is in equilibrium, that means for every state less than a_1 the sender would prefer the guess $a_1/2$ to the guess $a_1 + 1/2$. Similarly for every state greater than a_1 the send prefers guess $a_1 + 1/2$ to guess $a_1/2$.

e. Since the payoff function is continuous, these two conditions also entail that at state a_1 the sender is indifferent between the two guesses. This allows us to calculate the value of a_1

f. Namely,

$$-(a_1 - 0.1 - a_1/2)^2 = -(a_1 - 0.1 - (a_1 + 1)/2)^2$$

g. We can solve for a_1 and we get $a_1 = 0.3$.

iii. 3-partition case

a. We can do the same to find a three partition case

b. Now the sender seperates at two points a_1 and a_2, and we can find their location using the same method

c. Except we find there is no such parition. The equations do not have solutions in $(0, 1)$.

d. So there is no 3-partition equilibrium. And by extension, no 4, 5, 6, ...