Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Please do not use any sources on the web or in textbooks or journals. Work out the problem on your own.

Problem 1

Consider a game in normal form $\langle N, A, u \rangle$. We say that a strategy $x_i \in A_i$ is *strictly dominated* by another strategy $y_i \in A_i$ for player i, if for all opponent strategy profiles a_{-i}, $u_i(y_i, a_{-i}) > u_i(x_i, a_{-i})$. That is, y_i always does better than x_i no matter what the other opponents do. One creates a reduced game by removing all those strategies which are strictly dominated for a player. This removal might have produced new strategies which are strictly dominated, one can now remove those. Etc. This is called *iterated deletion of strictly dominated strategies.*

Part A

Use iterative deletion of *strictly* dominated strategies to reduce this game to one strategy for each player

	l	m	r
T	73, 25	57, 42	66, 32
M	80, 26	35, 12	32, 54
B	28, 27	63, 31	54, 29

Show that the resulting strategy pair is a Nash equilibrium of the original game. Is it the only Nash equilibrium?

Part B

Suppose an arbitrary game (with an arbitrary finite number of players) where you have iteratively deleted strictly dominated strategies and you are now left with only one strategy for each player. Is it the remaining strategy profile a Nash equilibrium? If the answer is yes, prove that for every game that can be solve in this way the remaining strategy profile is a Nash equilibrium. If the answer is no, give an example.

Part C

Suppose an arbitrary game (with an arbitrary finite number of players) that has been reduced by iterated deletion of strictly dominated strategies. Is it possible that a Nash equilibrium of the original game has been eliminated by iterated deletion? If so, give an example. If not, prove this isn't possible

Problem 2

A strategy x_i is weakly dominated by y_i if for all opponent strategy profiles a_{-i}, $u_i(y_i, a_{-i}) \geq u_i(x_i, a_{-i})$ and for *at least one* opponent strategy profile a'_{-i}, $u_i(y_i, a'_{-i}) > u_i(x_i, a'_{-i})$. A weakly dominated strategy is one that is never better and sometimes worse than the strategy which dominates it. We can also think of iterated deletion of weakly dominated strategies along the same lines as above.

Part A

Suppose an arbitrary game (with an arbitrary finite number of players) where you have iteratively deleted weakly dominated strategies and you are now left with only one strategy for each player. Is it the remaining strategy profile a Nash equilibrium? If the answer is yes, prove that for every game that can be solve in this way the remaining strategy profile is a Nash equilibrium. If the answer is no, give an example.

Part B

Suppose an arbitrary game (with an arbitrary finite number of players) that has been reduced by iterated deletion of weakly dominated strategies. Is it possible that a Nash equilibrium of the original game has been eliminated by iterated deletion? If so, give an example. If not, prove this isn't possible

Problem 3

Recall the result (due to Pearce) that we studied together:

NEVER BEST REPLIES ARE DOMINATED: In a finite decision problem with no moral hazard, if and only if an option o has no Bayes-model that is, if and only if for each probability distribution on the states of uncertainty option o fails to maximize expected utility then there is a mixed strategy alternative p^* that simply dominates option o in the partition of the states.

Now, consider a finite, normal form (simultaneous play), three person game, with players Row, Column, and Matrix.

- Row-player chooses one of the two rows, { *UP, DOWN* }.
- Column-player chooses one of the two columns { *LEFT, RIGHT* }.
- Matrix-player chooses one of four 2×2 Matrices { *A, B, C, D* }.

Problem 3 continued on next page. . .

The cardinal utility payoffs within each box of these 2×2 matrices are the same for all three players. For example, if Row plays *UP*, Column plays *LEFT*, and Matrix plays Matrix A, each player receives 9 cardinal units.

	LEFT	*RIGHT*
UP	9	0
DOWN	0	0

Matrix A

	LEFT	*RIGHT*
UP	0	9
DOWN	9	0

Matrix B

	LEFT	*RIGHT*
UP	0	0
DOWN	0	9

Matrix C

	LEFT	*RIGHT*
UP	6	0
DOWN	0	6

Matrix D

Part A

Verify that when Row-player uses a mixed strategy p:(1-p) for *UP:DOWN*, and when Column-player uses a mixed strategy q:(1-q) for *LEFT:RIGHT*, then the option Matrix D is never a best-reply.

Hint: Recall that the players strategies are jointly probabilistically independent. Regarding Matrix-players choices, verify each of the following:

- Matrix A is strictly better than Matrix D when $pq/(1-p)(1-q) > 2$ (case 1)
- Matrix C is strictly better than Matrix D when $(1-p)(1-q)/pq > 2$ (case 2)
- Matrix B is strictly better than Matrix D when neither case 1 nor case 2 obtains.

Part B

Verify that there is no mixed strategy involving the three options Matrix A, Matrix B, and Matrix C (with respective probabilities p_A, p_B, p_C, with $p_A + p_B + p_C = 1$) that dominates Matrix D in the four element partition { *UP-LEFT; UP-RIGHT; DOWN-LEFT; DOWN-RIGHT* }.

Part C

Explain in detail why the claims from Parts 1 and 2 do not constitute a counterexample to the result that NEVER BEST REPLIES ARE DOMINATED.

Hint: What does Pearces result say about the 4-option, 4-state decision problem that MATRIX player has: with partition { *UP-LEFT; UP-RIGHT; DOWN-LEFT; DOWN-RIGHT* } as the 4 states, and with menu { Matrix A, Matrix B, Matrix C, Matrix D } as the 4 (pure) options. How is this decision problem different from the situation Matrix-player faces in the 3-person, normal form game above?