

Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. **IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.**

Please do not use any sources on the web or in textbooks or journals. Work out the problem on your own.

## Problem 1

Suppose a game like the Stag Hunt, but with more players. Suppose that there are  $m$  hunters, but only  $n$  need to cooperate to catch the stag. (Assume  $2 \leq n < m$ .) If a stag is caught, the stag is divided equally among those who hunted stag. Like in the original game, a hunter who hunts hare gets a hare for sure which is worth 1.

### Part A

Assume that the Stag is worth  $x$  where  $x > m$ . What are the pure strategy Nash equilibria of this game?

### Part B

What about the case where  $m > x > n$ ?

## Problem 2

Consider a game  $\langle N, A, u \rangle$ . We say that a strategy  $x_i \in A_i$  is *strictly dominated* by another strategy  $y_i \in A_i$  if for all opponent strategy profiles  $a_{-i}$ ,  $u_i(y_i, a_{-i}) > u_i(x_i, a_{-i})$ . That is,  $y_i$  always does better than  $x_i$  no matter what the other opponents do. One can reduce a game by iteratively removing these strategies. One creates a reduced game by removing all those strategies which are dominated for a player. This removal might have produced new strategies which are dominated, one can now remove those. Etc. This is called *iterated deletion of strictly dominated strategies*.

A strategy  $x_i$  is weakly dominated by  $y_i$  if for all opponent strategy profiles  $a_{-i}$ ,  $u_i(y_i, a_{-i}) \geq u_i(x_i, a_{-i})$  and for *at least one* opponent strategy profile  $a'_{-i}$ ,  $u_i(y_i, a'_{-i}) > u_i(x_i, a'_{-i})$ . A weakly dominated strategy is one that is never better and sometimes worse than the strategy which dominates it.

### Part A

Use iterative deletion of *strictly* dominated strategies to reduce this game to one strategy for each player

	l	m	r
T	73, 25	57, 42	66, 32
M	80, 26	35, 12	32, 54
B	28, 27	63, 31	54, 29

Show that the resulting strategy pair is a Nash equilibrium of the original game. Is it the only Nash equilibrium?

**Part B**

For an arbitrary game, prove that if after iteratively eliminating *strictly* dominated strategies you are left with only one strategy for each player that this strategy is a Nash equilibrium. Prove the same for iterated elimination of *weakly* dominated strategies.

**Part C**

Can iterated deletion of strictly dominated strategies remove any Nash equilibria? (I.e. are there Nash equilibria which involve strategies that are eliminated by iterative deletion of dominated strategies?) Either give a counter example or a proof. What about iterative deletion of weakly dominated strategies?

**Problem 3**

In Santa Fe there is a bar named El Farol. On a given night suppose that three groups of friends are considering going to El Farol (treat each group like it is a single player). El Farol is a cool bar, but it's a little too small to hold all three groups. If a group goes to El Farol and there are two or fewer groups there, the payoff from going is 2. If however all three go the payoff is  $-1/2$ . The utility for staying home for each group is 0.

What are the (pure and mixed) Nash equilibria in this game? Which equilibria are *socially optimal* – that is they maximize the sum of the utilities? Is there anything that makes the socially inferior Nash equilibria appealing?

**Problem 4**

A strict Nash equilibrium is a Nash equilibrium where unilateral deviations by any player will make that player strictly worse off – no player is indifferent between performing her part of the equilibrium and some other action. Are any mixed strategy Nash equilibria strict? If you answer yes, give an example. If you answer no, prove why no mixed strategy Nash equilibria can be strict.

**Problem 5**

A two player symmetric game is a game  $\langle \{1, 2\}, A, u \rangle$  where  $A_1 = A_2$  and  $u_1(\langle i, j \rangle) = u_2(\langle j, i \rangle)$ . That is, it doesn't matter the identity of the player only the strategies played. (The Prisoner's dilemma, the stag hunt, and chicken are all symmetric; matching pennies is not.) A Nash equilibrium  $a^*$  is symmetric if  $a^* = \langle a, a \rangle$ . That is both players are playing the same strategy.

Show that, when we consider mixed strategies, every two-strategy two-player symmetric game has a symmetric Nash equilibrium. For extra credit show this fact obtains for any finite two-player symmetric game. What about an  $n$  player symmetric game?