

Problem 1

Recall the general definition of a symmetric 2x2 game:

| | | |
|---|-----|-----|
| | U | D |
| U | a | b |
| D | c | d |

Consider the Coordination class, where $a > c$ and $d > b$. Prove that all trajectories in the continuous time replicator dynamics converge to a Nash equilibrium of the game.

Problem 2

Naively people often talk about natural selection as producing what is good for the species. Natural selection in strategic situations does not always behave this way. Please provide an example (and prove that the example fits) where:

- (a) All interior trajectories in the continuous time replicator dynamics feature increasing average utility throughout the trajectory
- (b) All interior trajectories in the replicator dynamics feature decreasing average utility throughout the trajectory
- (c) For extra-credit, some interior trajectories result in increasing and others in decreasing average utility

Problem 3

The Nash Bargaining game has been used to model the evolution of norms of fairness. In this simplified version of the Nash bargaining game, each player chooses a demand (either $1/3$, $1/2$, or $2/3$). If the two demands do not sum to more than one, each player gets her demand. If the two demands do sum to more than one, both players get 0. This is a symmetric 3-strategy game. Please do the following

- (a) Find all the (pure and mixed) Nash equilibria of the game
- (b) Determine which (if any) of these equilibria are asymptotically stable in the continuous time replicator dynamics. (I would recommend using the Jacobian method, although I'd be eager to see a use of the Lyapunov function method if you could figure one out.)