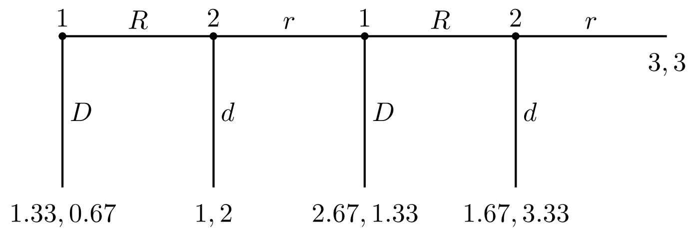
Problem 1

Consider this game called "the centipede game" that I mentioned at the beginning of class this morning.



Part A What is the normal form representation of this game?

Part B What are the pure strategy Nash equilibria of this game?

Part C Which Nash equilibria are subgame perfect?

Problem 2

Consider the game played by Paul Revere and the sexton of the Old North Church. The British decide whether they are invading by land or by sea. Let's not worry about them too much, assume they flip a coin to decide what to do.

The sexton can see whence the troops arrive and can do one of two things: hang one lantern or hang two lanterns. Paul Revere can observe what the sexton has done, but cannot observe the British. Paul Revere can then do one of two things: he can ride through the city warning the locals that the British are coming by land or that they are coming by sea. Assume that if Paul correctly informs the city both Paul and Sexton get a payoff of 1, 0 otherwise.

What are the Nash equilibria of this game?

Problem 3

Consider a job search game. There are two types of potential employees *high quality* and *low quality.* Nature decides with probability p if the employee if of *high quality.* The employee can then choose either to go to college or not. In order to go to college the employee must pay a cost c, if she doesn't go to college she pays no cost. The employer observes whether the employee went to college (but not the quality) and then must determine whether to hire the employee or not. If the employer hires the employee, the employee gets paid 1 (regardless of type), otherwise the employee gets 0. If the employer hires the *high quality* employee he gets a payoff of 1, if the employer hires the *low quality* employee he gets a payoff of -1, and if the employer doesn't hire he gets a payoff of 0.

Describe all the pure strategy Nash equilibria of this game for all values of p and c.

Problem 4

Consider a variant on the ultimatum game. Carlos and Shannon will split a dollar. Carlos moves first and proposes a split to Shannon (he chooses $x \in [0, 1]$). Shannon observes Carlos' offer and has two options: she can accept in which case she gets $(1 - x)$ and Carlos gets x or she can reject. If she rejects she makes a counterproposal to Carlos, but because they are taking too long the pie shrinks (formally, Shannon chooses a $y \in [0, \delta]$ where $0 < \delta < 1$). Carlos can then either accept Shannon's counterproposal in which case Carlos gets $(\delta - y)$ and Shannon gets y or he can reject. If Carlos rejects they both get 0.

What are the subgame perfect equilibria of this game? (You may make a few simplifying assumptions about what Carlos and Shannon do when indifferent or the general properties of their strategies if you think it might help.)

Very difficult

Suppose this process continues indefinitely. Carlos if he rejects can make a counterproposal in $[0, \delta^2]$ and then Shannon can make a counter proposal in $[0.\delta^3]$, etc. What are the subgame perfect equilibria here?

Problem 5

Suppose the following game. There is a pile of N stones and two players. The players alternate taking 1, 2 or 3 stones from the pile. The person who removes the last stone loses and the other player wins. For what values of N does player 1 have a winning strategy and for what values of N does player 2 have a winning strategy? Prove your answer.