Problem 1

Remember Matching pennies:

	H	T
h	1 -1	-1 1
t	-1 1	1 -1

Are there any ESS in this game? If so what are they?

Problem 2

Consider this game:

	A	B	C	D
A	3 3	2 4	4 2	0 0
B	4 2	3 3	2 4	0 0
C	2 4	4 2	3 3	0 0
D	0 0	0 0	0 0	1 1

Part A

What are all the Nash equilibria (pure and mixed) of this game?

Part B

What are the ESS of this game? What is your reaction to the ESS? Does it seem plausible as the result of evolution?

Problem 3

Often people speak informally about natural selection resulting in what is best for the species. In this problem, you will illustrate several errors with this conception of evolution (at least so far as it relates to Evolutionarily Stable Strategies).

Consider a symmetric two-player game. Suppose there is a population composed of types each of which plays one of the pure strategies in the game. The "population strategy" is like a mixed strategy – a list of proportions playing each individual pure strategy. Let the population strategy be $\mathcal{P} =< p_1, p_2, \ldots, p_n >$. Assume that players are randomly paired to play the game, so the payoff for an individual pure strategy type is given by the payoff of that type against $\mathcal{P}$. The payoff of the population is calculated by using $\mathcal{P}$ to construct an average payoff based on the payoffs to each of the pure strategies.

For this problem, you should construct three games each of which has a single unique Evolutionarily Stable Strategy. Construct the first game so that the payoff to the population is higher in the ESS than it is in any other state. Construct the second game so that the payoff to the population is lower in the ESS than it is in any other state. And finally, construct a game where the payoff in ESS is higher than some states and lower than others.

Problem 4

For students with some differential equations background.

Consider the Prisoner's dilemma

	C	D
C	3, 3	0, 4
D	4, 0	1, 1

We can represent the evolution of a population of critters playing the Prisoner's dilemma with a single differential equation:

$$\dot{x} = x(u(C, c) - u(x, x))$$

Where x represents the frequency of cooperators in the population. $u(C, x) = 3x$ and $u(x, x) = 3x^2 + 4x(1 - x) + (1 - x)^2$.

What are the fixed points of this differential equation (those points where the change in x is zero)? Which of these points are asymptotically stable?

Problem 5

A strict Nash equilibrium is a Nash equilibrium where unilateral deviations by any player will make that player strictly worse off – no player is indifferent between performing her part of the equilibrium and some other action. Are any mixed strategy Nash equilibria strict? If you answer yes, give an example. If you answer no, prove why no mixed strategy Nash equilibria can be strict.

Problem 6

Difficult

A game is said to have a *non-trivial extensive form* if there is no way to represent the game where each player has only one information set (where they make a decision with at least two options). Effectively, this means that at least one player acts later than another and is able to observe the actions of that other player before acting. Prove that every Nash equilibrium a game with a non-trivial extensive form and no moves by nature is *not* strict. That is, in every game with a non-trivial extensive form, one player *must* be indifferent between pursuing her part of the Nash equilibrium and some other strategy

What does this say about ESS in these games? Can non-trivial extensive form games (without moves by nature) have an ESS?