Problem 1

Find all the pure strategy Nash equilibria for the following game:

	l	c	r
T	4, 4	2, 3	1, 1
M	3, 1	3, 5	4, 6
B	1, 8	3, 8	2, 1

Problem 2

Part A Find all the pure strategy Nash equilibria for the following game:

	H	T
h	7, 3	4, 4
t	3, 4	5, 0

Part B

Find all the mixed strategy Nash equilibria.

Problem 3

Part A

Use iterative deletion of strictly dominated strategies to reduce this game to one strategy for each player

	l	m	r
T	73, 25	57, 42	66, 32
M	80, 26	35, 12	32, 54
B	28, 27	63, 31	54, 29

Show that the resulting strategy pair is a Nash equilibrium of the original game. Is it the only Nash equilibrium?

Part B - Difficult

For an arbitrary game, show that if after iteratively eliminating strictly dominated strategies you are left with only one strategy for each player that this strategy is a Nash equilibrium. Show the same for iterated elimination of weakly dominated strategies.

Part C - More difficult

Can iterated deletion of strictly dominated strategies remove any Nash equilibria? (I.e. are there Nash equilibria which involve strategies that are eliminated by iterative deletion of dominated strategies?) Either give a counter example or a proof. What about iterative deletion of weakly dominated strategies?

Problem 4

Suppose two people are dividing a dollar. Each proposes an amount of the dollar she would like. If the two demands are compatible, i.e. that add up to no more than a dollar, then each individual gets what she demands. If, however, they add up to more than a dollar, neither gets anything. What are the Nash equilibria of this game? Do any of them strike you as particularly interesting?

Problem 5

Suppose two neighbors agree to improve a piece of shared property behind their houses. Each neighbor has a budget b of which she can spend any amount x_i on the shared property ($0 \leq x_i \leq b$). What ever is not spent on the shared property is spent on the individual's own property. Assume each dollar spent improves the property by an equal amount, and each neighbor cares about the quality of the joint property slightly less than her own. In particular Suppose neighbor 1's utility function looks like this:

$$\frac{3}{4}(x_1 + x_2) + (b - x_1)$$

(2's is the same but with the number reversed.)

What are the Nash equilibria of this game? Is this the best outcome of the game for each player?

Problem 6

In class we discussed the "Guess 2/3 of the average" game. Now, I'd like you to solve the "Guess the average game." There are n players and each player guesses a real number in $[0, 1]$. The players who are closest to the average of all the guesses equally split a prize. What are all the pure strategy Nash equilibria for this game?

Problem 7

Suppose a game like the Stag Hunt, but with more players. Suppose that there are m hunters, but only n need to cooperate to catch the stag. (Assume $2 \leq n < m$.) If a stag is caught, the stag is divided equally among those who hunted stag. Like in the original game, a hunter who hunts hare gets a hare for sure.

Part A

Assume that the Stag is worth x where $x > m$. What are the Nash equilibria of this game?

Part B

What about the case where $m > x > n$?