Written answers are acceptable so long as they are legable. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Hawk-Dove

Suppose two animals come upon a resource which is worth V to each of them. They each have two strategies at their disposal, they can either escalate to a conflict (*Hawk*) or back down (*Dove*). If one escalates and the other does not, the one who escalates gets the resource and the other gets nothing. If they both back down, they share the resource equally. If they both escalate, they fight for the resource and one of them wins the resource with probability 0.5. However the fight extracts a cost C from both of them.

What strategies of this game are equilibria depends on the relative values of V and C. Identify what the different possible equilibria of this game are and what is required (in terms of V and C) for that to be an equilibrium.

Problem 2

Consider the game of chicken pictured here:

	Hawk	*Dove*
Hawk	0, 0	3, 1
Dove	1, 3	2, 2

From what starting strategies will the myopic best response dynamics (Cournot adjustment dynamics) converge to a Nash equilibrium? From what starting strategies will it not converge to a Nash equilibrium? If there are any states that don't converge, what will happen in those cases?

Problem 3

Suppose that there are three candidates (Alice, Bob, and Carol) for a political office and three voters that decide who wins. Each of them has a strict preference over the different candidates, that is each has a favorite that they prefer over their second favorite over their least favorite. (Each voter can have different preferences from the others, of course.) Suppose the voting proceeds as follows. First the three voters vote on whether they would prefer Alice or Bob. Whoever secures a majority then is pitted against Carol. Whoever wins this vote is promoted to the political office.

Show that *regardless of the preferences of the individual voters* there are Nash equilibria where Alice wins, where Bob wins, and where Carol wins.

Problem 4

Often people speak informally about natural selection resulting in what is best for the species. In this problem, you will illustrate several errors with this conception of evolution (at least so far as it relates to Evolutionarily Stable Strategies).

Consider a symmetric two-player game. Suppose there is a population composed of types each of which plays one of the pure strategies in the game. The "population strategy" is like a mixed strategy – a list of proportions playing each individual pure strategy. Let the population strategy be $\mathcal{P} =< p_1, p_2, \ldots, p_n >$. Assume that players are randomly paired to play the game, so the payoff for an individual pure strategy type is given by the payoff of that type against $\mathcal{P}$. The payoff of the population is calculated by using $\mathcal{P}$ to construct an average payoff based on the payoffs to each of the pure strategies.

For this problem, you should construct three games each of which has a single unique Evolutionarily Stable Strategy. Construct the first game so that the payoff to the population is higher in the ESS than it is in any other state. Construct the second game so that the payoff to the population is lower in the ESS than it is in any other state. And finally, construct a game where the payoff in ESS is higher than some states and lower than others.

Graduate student problems (extra credit for undergrads)

Problem 5

The dollar auction

I will be auctioning off a crisp, new one dollar bill to some number of bidders. The bid starts at \$0.01, two bidders will take turns deciding whether or not they are willing to up the bid by one cent or give up the auction to the other bidder. When a player gives up the auction ends and *both* players must pay me the highest bid they made. So if one player bid \$0.51 and the other bid \$0.50 but refused to bid \$0.52, one player pays me \$0.51 and the other pays me \$0.50. The player with the highest bid wins the crisp, new one dollar bill.

Suppose that both players have an infinite amount of money, and so we have an infinite game. Are there any pure strategy Nash equilibria? If so, what are they? If not, show that there are not any.

Suppose both players are perpetually optimistic, they both think at each stage that the other player will be unwilling to go one higher. What would happen?

Problem 6

In voting games like the one in problem 3, a voter is said to be voting *strategically* if her strategy commits her to voting for a candidate she disprefers over a candidate she prefers. A voter is voting *straightforwardly* if she is not voting strategically.

Using the game described in problem 3, illustrate preferences on the part of voters such that straightforward voting is *not* a Nash equilibrium of the game. Give a Nash equilibrium for the preferences you gave (that is different from the ones in problem 3).