

Written answers are acceptable so long as they are legable. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Find all the Nash equilibria of this game.

	A	B				
A	<table><tr><td>2</td></tr><tr><td>3</td></tr></table>	2	3	<table><tr><td>0</td></tr><tr><td>0</td></tr></table>	0	0
2						
3						
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B	<table><tr><td>1</td></tr><tr><td>1</td></tr></table>	1	1	<table><tr><td>2</td></tr><tr><td>2</td></tr></table>	2	2
1						
1						
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2						

Problem 2

Consider the Prisoner's dilemma pictured here:

	<i>Dove</i>	<i>Hawk</i>								
<i>Dove</i>	<table><tr><td>3</td><td>3</td></tr><tr><td>3</td><td>1</td></tr></table>	3	3	3	1	<table><tr><td>4</td><td>1</td></tr><tr><td>4</td><td>2</td></tr></table>	4	1	4	2
3	3									
3	1									
4	1									
4	2									
<i>Hawk</i>	<table><tr><td>1</td><td>2</td></tr><tr><td>4</td><td>2</td></tr></table>	1	2	4	2	<table><tr><td>2</td><td>2</td></tr><tr><td>2</td><td>2</td></tr></table>	2	2	2	2
1	2									
4	2									
2	2									
2	2									

Draw the cooperative payoff region for this game. If different, draw the uncooperative payoff region. If it's the same, show how you came to the conclusion they were the same.

Problem 3

Sealed bid, Second price auction

In sealed bid, second price auctions every individual submits just one bid without knowing what the bids of the other people are. The winner is the person who submits the highest bid, but she only has to pay the value of the bid of the *second* highest bidder. So if the bids are \$10, \$8, and \$5. The person who bid \$10 pays \$8 for the item.

Suppose that each individual has a distinct value for the item, represented by v_i where i represents the bidder. Each individual chooses a bid, b_i for the item. If they win they get the item which is equivalent to them getting v_i , but they have to pay the second highest bid so their payoff is the difference between v_i and the amount they paid. Everyone else gets nothing and pays nothing, so their payoffs are zero.

Show that the only strategies which survive iterated elimination of weakly dominated strategies are those where each individual bids v_i , their value for the item.

Problem 4

There is a bar in Sante Fe called the “El Farol” which has become the namesake of a famous game theory problem. The El Farol can be a fun bar, but if it gets too crowded it’s not worth going to. Suppose there are three potential customers and each must decide if she wants to go to the El Farol. If less than three people choose to go, then those that go get a payoff of 2. If all three people go, then they get a payoff of -1. Not going to the bar gives you a payoff of 0 regardless of the number of people that go to the bar.

Find *all* the Nash equilibria of this game.

Graduate student problems (extra credit for undergrads)

Problem 5

Is there any game with a strictly dominated strategy for a player where that strategy is played with positive probability in a mixed strategy Nash equilibrium? If so, illustrate a case. If not, prove it’s impossible.

Do the same for weakly dominated strategies.

Problem 6

Consider this very simple version of poker. Two players begin by anteing \$2 into a central “pot”. Player one is dealt a card that is either “High” or “Low” with equal probability. Player 2 is not dealt a card (suppose her card is “Medium” for sure). Player 1 looks at his own card and then decides whether or not to bet \$2 or fold. If he folds, the game is over and Player 2 wins the pot. If he bets, Player 2 decides whether to call (by putting in \$2) or fold. If Player 2 folds, the game is over and Player 1 wins.

If Player 2 calls, Player 1 again is given the opportunity to bet (\$2 again) or fold. If he folds, Player 2 wins the pot. If he bets, Player 2 has to choose to call or fold again. If Player 2 folds, Player 1 wins. If Player 2 calls then Player 1 wins if he has the “High” card and loses if he has the “Low” card.

Find the mixed strategy Nash equilibrium of this game.