Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Please do not use any sources on the web or in textbooks or journals. Work out the problem on your own.

Problem 1

In class we discussed the "Guess 2/3 of the average" game. Now, I'd like you to solve the "Guess the average game." There are n players and each player guesses a real number in $[0, 1]$. The players who are closest to the average of all the guesses equally split a prize. What are all the pure strategy Nash equilibria for this game?

Problem 2

Consider the following game:

	C	D
A	0,0	2, 1
B	1, 1	0, 0

Find all of the pure and mixed strategy Nash equilibria of this game.

Problem 3

Consider a game $\langle N, A, u\rangle$. We say that a strategy $x_i \in A_i$ is *strictly dominated* by another strategy $y_i \in A_i$ if for all opponent strategy profiles a_{-i}, $u_i(y_i, a_{-i}) > u_i(x_i, a_{-i})$. That is, y_i always does better than x_i no matter what the other opponents do. One can reduce a game by iteratively removing these strategies. One creates a reduced game by removing all those strategies which are dominated for a player. This removal might have produced new strategies which are dominated, one can now remove those. Etc. This is called *iterated deletion of strictly dominated strategies.*

A strategy x_i is weakly dominated by y_i if for all opponent strategy profiles a_{-i}, $u_i(y_i, a_{-i}) \geq u_i(x_i, a_{-i})$ and for *at least one* opponent strategy profile a'_{-i}, $u_i(y_i, a'_{-i}) > u_i(x_i, a'_{-i})$. A weakly dominated strategy is one that is never better and sometimes worse than the strategy which dominates it.

Part A

Use iterative deletion of *strictly* dominated strategies to reduce this game to one strategy for each player

	l	m	r
T	73, 25	57, 42	66, 32
M	80, 26	35, 12	32, 54
B	28, 27	63, 31	54, 29

Show that the resulting strategy pair is a Nash equilibrium of the original game. Is it the only Nash equilibrium?

Problem 3 continued on next page...

Part B

For an arbitrary game, prove that if after iteratively eliminating *strictly* dominated strategies you are left with only one strategy for each player that this strategy is a Nash equilibrium. Prove the same for iterated elimination of *weakly* dominated strategies.

Part C

Can iterated deletion of strictly dominated strategies remove any Nash equilibria? (I.e. are there Nash equilibria which involve strategies that are eliminated by iterative deletion of dominated strategies?) Either give a counter example or a proof. What about iterative deletion of weakly dominated strategies?

Problem 4

In Santa Fe there is a bar named El Farol. On a given night suppose that three groups of friends are considering going to El Farol (treat each group like it is a single player). El Farol is a cool bar, but it's a little to small to hold all three groups. If a group goes to El Farol and there are two or fewer groups there, the payoff from going is 2. If however all three go the payoff is -1/2. The utility for staying home for each group is 0.

What are the (pure and mixed) Nash equilibria in this game? Which equilibria are *socially optimal* – that is they maximize the sum of the utilities? Is there anything that makes the socially inferior Nash equilibria appealing?

Problem 5

A two player symmetric game is a game $\langle \{1, 2\}, A, u\rangle$ where $A_1 = A_2$ and $u_1(\langle i, j\rangle) = u_2(\langle j, i\rangle)$. That is, it doesn't matter the identity of the player only the strategies played. (The Prisoner's dilemma, the stag hunt, and chicken are all symmetric; matching pennies is not.) A Nash equilibrium a^* is symmetric if $a^* = \langle a, a\rangle$. That is both players are playing the same strategy.

Show that, when we consider mixed strategies, every two-strategy two-player symmetric game has a symmetric Nash equilibrium. For extra credit show this fact obtains for any finite two-player symmetric game. What about an n player symmetric game?